Nucleus

Towards a Unified Dynamics Solver for Computer Graphics

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Motivation

Ruysdael (1628-1682)
Too many Solvers

rigid bodies  cloth  fluids

Solvers have to interact
Unified Solver

Two-way interaction
Unified Solver

King Dome, Seattle, March 26, 2000
History

Simple idea : 2000
Fooling around (palm demo) : 2001
First prototypes : 2003-2005
First real implementation : 2005
First public demo : 2006
Released in MAYA 8.5 (nCloth) : 2007
nParticles : 2008
Nucleus

Nucleus

API

MAYA
Nucleus

~ 100 files

MAYA

~ 40,000 files
Approach

General Shape Model

Stable Dynamics
Shape Model

“Simplicial Approximation Theorem”

1. Theorem (Brouwer 1910)

Every continuous mapping can be approximated by a piece-wise linear simplicial map.

Fundamental theorem of CG modeling
1 Theorem

Let $K$ and $L$ be complexes; let $K$ be finite. Given a continuous map $h : |K| \to |L|$ there is an $N$ such that $h$ has a simplicial approximation $f : \text{sd}^N K \to L$. 
Simplicial Complex
Simplicial Complex
Simplicial Complex

Dover: $2.75
Shape Model

2 edges:
- (8,9) + (7,8)

7 triangles:
+ (0,1,4), + (1,2,5)
- (0,3,4), - (1,4,5)
+ (3,4,7), + (4,5,6),
+ (4,6,7)

(i,j,k) = -(i,k,j)

Definition purely topological
Simplicial Complex

Single class for all primitives:

class simplex {
    int k;
    int sign;
    int vertex[k+1];
    int child[k+1];
    int n_parents;
    int parent[n_parents];
};

Code not specific for a primitive: simple
Dynamics

\[ \mathbf{x}(t) = (x_1(t), \cdots, x_N(t)) \in \mathbb{R}^{3N} \]
Dynamics
Newton’s Law

\[ \ddot{x} = -\nabla f(x) + f_e \]

\[ x(0) = x_0 \]

\[ \dot{x}(0) = v_0 \]

\[ E = \frac{1}{2} \dot{x}^2 + f(x) - f_e \cdot x \]
Isaac Newton

Isaac Newton's *Principia* 1687
Simple Example
Spring

\[ \ddot{x} = -x \]

\[ x(0) = x_0 \]

\[ \dot{x}(0) = v_0 \]

\[ E = \frac{1}{2} \dot{x}^2 + \frac{1}{2} x^2 \]
Spring

\[ \ddot{x} = -x \]

\[ z = x + i \dot{x} \quad \in \mathbb{C} \]

\[ \dot{z} = -i z \]

\[ z(0) = z_0 \]
Spring

\[ \dot{z} = -i \ z \]

\[ z(0) = z_0 \]

\[ z(t) = e^{-it} \ z(0) \]
A Thought...

$\mathbb{R}^{6n} \rightarrow \mathbb{C}^{3n}$
Spring

$|z(t)|^2 = |z(0)|^2$
Spring

\[ \dot{z} = -i z \]
Explicit Solver
Explicit Solver
Implicit Solver
Explicit Solver

\[
\frac{1}{\hbar} (z^n - z^{n-1}) = -i z^{n-1}
\]

\[
z^n = z^{n-1} - i\hbar z^{n-1}
\]

\[
z^n = (1 + \hbar^2) e^{-i\hbar} z^{n-1}
\]

\[
z^n = (1 + \hbar^2)^n e^{-inh} z^0
\]
Implicit Solver

\[ \frac{1}{h} (z^n - z^{n-1}) = -i z^n \]

\[ (1 + ih) z^n = z^{n-1} \]

\[ z^n = \frac{1}{1 + h^2} e^{-ih} z^{n-1} \]

\[ z^n = \frac{1}{(1 + h^2)^n} e^{-inh} z^0 \]
Symplectic Solver
Symplectic Solver
Symplectic Solver
Symplectic Solver
Symplectic?

“Official Definition”:

“Plaiting or joining together; -- said of a bone next above the quadrate in the mandibular suspensorium of many fishes, which unites together the other bones of the suspensorium”.

Fish bones?
Symplectic?

In math: Hermann Weyl (1930’s) : com-plex → sym-plectic

Com : Latin root
Sym : Greek root
Symplectic?
Symplectic?

\[
\begin{pmatrix}
\dot{x}^1 \\
\dot{x}^1
\end{pmatrix} = 
\begin{pmatrix}
1 & -h \\
h & 1 - h^2
\end{pmatrix}
\begin{pmatrix}
\dot{x}^0 \\
x^0
\end{pmatrix}
\]

A
Demo

Spring Demo

•——•——•——•——•——•——•——•——
Simple Idea
Simple Idea
Nucleus

Strategy:

- Use constraints
- Implicit on velocity
- Explicit on position
Nucleus

\[ C (x + v + \delta v) = 0 \]

Solve for: \( \delta v \)

\[ v = v + \delta v \]
\[ x = x + v \]
Deformations

1

2

3
Deformations

1-simplex

2-simplex

3-simplex

1/2

1/3
How To Solve?

\[ C \left( x + v + \delta v \right) = 0 \]

Non-linear...

One constraint at a time
Linearize
BFGS
One Constraint

\[ f_k(t) = C_k \left( x + v + t \, d_k \right) = 0 \]

\[ f_k(t) \approx f_k(0) + t \, f'_k(0) = 0 \]

\[ t = -\frac{f_k(0)}{f'_k(0)} \]
Search Direction

What about $d_k$?
Search Direction

Bridson, Marino and Fedkiw SCA 2003
Linearize

\[ \nabla C (x + v) \delta v = -C (x + v) \]

\[ Au = b \quad 3n \times m \]

\[ AAT v = b \quad u = A^T v \]

LSQR, CGLS
BFGS

\[ C(x + v + \delta v) \]
\[ \nabla C(x + v + \delta v) \]
\[ \delta v \]
One Constraint

Update velocity

Sequential vs Parallel

Gauss-Seidel vs Jacobi

Better convergence vs Slower convergence

Bias vs No bias

Hard to parallelize vs Easy to parallelize

Multigrid ?
parallel / no multigrid
sequential / no multigrid
parallel / with multigrid
sequential / multigrid
Stretch/Compression
Shear
Bending
Collisions

penalty vs space-time
Space-Time

time

space (1D)
Space-Time

time

dt

space (1D)
Space-Time

$V_t = a_t - b_t$

$V_0 < 0$

$V_1 > 0$

$V_t = a_t - b_t$

$V_0 < 0$

$V_t = a_t - b_t$

$V_0 < 0$

$V_1 > 0$

$V_t = a_t - b_t$
Space-Time

t = \frac{V_0}{(V_0 - V_1)}
Space-Time

elastic
Space-Time

inelastic
Space-Time

friction
Space-Time (2D)
Space-Time (2D)
Space-Time (2D)

Necessary but **not** sufficient
Space-Time (3D)
Space-Time (3D)
Summary

If \( V_0 \cdot V_1 > 0 \) stop

Find \( t \) such that \( V_t = 0 \)

Check if primitives overlap at \( t \)

If yes handle collision
Higher Dimensions?
Thickness
<table>
<thead>
<tr>
<th>Thickness</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic (2)</td>
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<tr>
<td>Quartic (4)</td>
<td><img src="quartic.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Sextic (6)</td>
<td><img src="sextic.png" alt="Diagram" /></td>
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</tbody>
</table>
Roots of a Polynomial

\[ t = \frac{1}{2a} \left( b \pm \sqrt{b^2 - 4ac} \right) \]

\( b^2 \gg 4ac \)
Roots of a Polynomial

\[ q = -b - \text{sgn}(b) \sqrt{b^2 - 4ac} \]

\[ t_1 = \frac{2c}{q} \]

\[ t_2 = \frac{q}{2a} \]
Roots of a Polynomial

No stable formulas for degrees 3 and 4.

No formulas for degree $> 4$ (Abel + Galois)
Roots of a Polynomial
Roots of a Polynomial
Roots of a Polynomial
Many Primitives

Use hierarchical data structures for speed

AABB tree (simple), actually kDOP
Many Primitives

Use hierarchical data structures for speed
Many Primitives

Use hierarchical data structures for speed
Many Primitives

Use hierarchical data structures for speed
Many Primitives
Many Primitives
Many Primitives
Many Primitives

Etc. Expensive in General
Many Primitives

Our approach:
Iterate over entire time step
Until all collisions resolved.

Avoids lockups and Zeno’s paradox
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Many Primitives
Demos

1D demo
(t key for space-time)

3D demo
Torture Tests
Torture Tests
Torture Tests
Torture Tests
Torture Tests
Torture Tests

Full render 1

Full render 2
Pressure Model

\[ PV (x + v + \delta v) = cM \]

- \( P \) : pressure
- \( M \) : mass
- \( V \) : volume
Pressure Model

No pressure
Pressure Model

Volume conservation
Pressure Model

Under pressure
Pressure Model

Under a lot of pressure
Pressure Model

Air tightness + Pump rate
Bend

Fully rendered
Bend

Fully rendered
Bend

Car crash
Fracture Animation
General Solver

Battle of the constraints

- Self-Collision
- Shear
- Bend
- Collision
- Stretch
General Solver

Collisions/stretch

Stretch/collisions
# General Solver v1.0

<table>
<thead>
<tr>
<th>Category</th>
<th>Count</th>
</tr>
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<tbody>
<tr>
<td>Bend (9)</td>
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<tr>
<td>Shear (7)</td>
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<tr>
<td>Stretch (26)</td>
<td></td>
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<tr>
<td>Self-Collisions (7)</td>
<td></td>
</tr>
<tr>
<td>Collisions (6)</td>
<td></td>
</tr>
</tbody>
</table>
General Solver

not interleaved

interleaved
|                  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Bend (9)         |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Shear (7)        |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Stretch (26)     |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Self-Collisions (7) |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Collisions (6)   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
## General Solver

|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Bend  |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Shear |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Stretch |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Self-Collisions |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Custom |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Collisions |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
Demos

Curve demo

Cloth demo
nParticles

New in MAYA 2009

Extends existing particles
nParticles
nParticles
nParticles

\[ \rho \left( x + v + \delta v \right) - \rho_0 = 0 \]

\[ \rho(x) = \sum_{i=1}^{N} m_i W \left( x - x_i \right) \]

Stable SPH
Solids
More Cloth Examples

Inflatable Girl

Inflatable Guy

Rigid bodies

ballerina

ballerina 2

Cloth drop
Duncan on the AREA


Brain
Future Work

Other nThings

Improve Collisions

Improve Constraint Solver
The End

Thank You